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T-R-402

Radar Scattering by Non-Spherical Particles

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TRANSLATION OF

RADAR SCATTERING BY NON-SPHERICAL PARTICLES

(Radiolokatsionnoe rasseianie
nesfericheskimi chastitsami)

by

A. B. Shupiatskii

Translated by

Robert Magruder

Tsentral'naia Aerologicheskaia Observatoriia, Trudy,
No. 30: 39-52, 1959.

This translation has been made by the
American Meteorological Society under
Contract AF 19(604)-6113, through the
support and sponsorship of the

AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
GEOPHYSICS RESEARCH DIRECTORATE
L. G. HANSCOM FIELD
BEDFORD, MASSACHUSETTS

1. "Radar scattering by non-spherical particles"
2. "Radiolokatsionnoe rasseianie nesfericheskimi chastitsami"
3. Shupiatskii, A. B. Tsentral'naia Aerologicheskaia Observatoriia, Trudy, No. 30: 39-52, 1959.
4. 23 typewritten pages
5. Date of translation: March 1963
6. Translator: Robert Magruder, Edited by David Kraus.
7. Translated for Air Force Cambridge Research Laboratories, Office of Aerospace Research, United States Air Force, L. G. Hanscom Field, Bedford, Massachusetts, by the American Meteorological Society, Contract number AF19(604)-6113.
8. Unclassified
9. Complete

RADAR SCATTERING BY NON-SPHERICAL PARTICLES

by

A. B. Shupiatskii

In theoretical works on the scattering and absorption of radio waves by precipitation and clouds, usually it is assumed that the meteorological particles are spherical. This assumption is completely justified for radiowave scattering by drop clouds and, in a number of cases, by raindrops as well. However, radar technique has now reached a stage where a radar signal can be detected from mixed and ice-crystal clouds. Thus, the study of the structure of radio waves reflected from ice crystals becomes a much more real problem. Further, experimental data obtained recently [3] have shown that large raindrops depart considerably from the spherical form.

For a strict solution of the problem of scattering and absorption of electromagnetic energy by a complex atmospheric particle, graphic conditions must be described which express the continuity of the field components at the particle surface. However, the wave equation can be broken down into variables and can be reduced to ordinary differential equations only for bodies of the simplest form. In particular, the wave equation can be solved for the case of reflection of electromagnetic waves from small spheroidal particles.

By small particles, we mean particles which are small compared with the wave length. This means that the particle dimension in any direction should not be larger than the size permitted by the Rayleigh law. Except for anomalously large cloud crystals, it may be assumed that all cloud particles and raindrops satisfy this condition for radiowaves of the centimeter band.

The actual particles of ice-crystal clouds are naturally more complex than spheroids. However, we can approach reality much more closely by approximating the rod-shaped and disk-shaped crystals with prolate and oblate spheriods, respectively, than we can by approximating them with spheres.

1. THEORY OF APPROXIMATION OF RADAR SCATTERING BY SMALL ELLIPSOIDAL PARTICLES

In a first approximation, Gans [1] examined scattering by ellipsoidal particles in connection with colloidal chemistry. Through his studies of the properties of suspensions, he established the spectral relationship between light scattered in a solution and the form of the particles suspended in the solution. We have used the results of Gans' work and analogous computations of the field components by L. D. Landau and E. M. Lifshits [2] in our study of the scattering of electromagnetic energy by atmospheric particles.

Let us represent the field potential outside the ellipsoid as the superposition of a field without the φ_0 particle and field φ' distorted by the presence of the ellipsoidal particles

$$\varphi = \varphi_0 + \varphi'. \tag{1}$$

At great distances from the ellipsoid, the unknown value of the distorted external field assumes the form of the field potential of the electric dipole.

In particular, for an ellipsoid with semiaxes a, b, c, which has dielectric permeability, the value of the dipole moment will be:

$$P_a = E_{oa}\frac{abc}{3}\frac{\varepsilon-1}{1+(\varepsilon-1)n_a}. \tag{2}$$

Here the n_a values and, analogously, in other directions the n_b and n_c values, depend only on the form of the ellipsoid and are determined by the expressions:

$$n_a = \frac{abc}{2}\int_0^\infty \frac{ds}{(s+a^2)R_s}, \quad n_b = \frac{abc}{2}\int_0^\infty \frac{ds}{(s+b^2)R_s}, \quad n_c = \frac{abc}{2}\int_0^\infty \frac{\partial s}{(s+c^2)R_s}. \tag{3}$$

The n_a, n_b, and n_c values have the following characteristics:

1) If a>b>c, then $n_a<n_b<n_c$, respectively; 2) the following equality obtains for any values of the semiaxes:

$$n_a + n_b + n_c = 1. \tag{4}$$

Elliptic integrals (3) are expressed by elementary functions only in the case of a biaxial spheroid. For a prolate spheroid with eccentricity $e=\sqrt{1-\frac{b^2}{a^2}}$ and semiaxes a>b = c, the first integral (3) is

$$n_a(e) = \frac{1-e^2}{2e^3}\left(\ln\frac{1+e}{1-e} - 2e\right), \tag{5}$$

and the second and the third are equal to one another by virtue of symmetry and can be determined from (4):

$$n_b = n_c = \frac{1}{2} - \frac{1}{2}n_a. \tag{6}$$

Correspondingly, for an oblate spheroid with eccentricity $e=\sqrt{1-\frac{c^2}{a^2}}$ and semiaxes a = b>c, the geometric factors are

$$n_c(e) = \frac{1}{e^2}\left(1-\sqrt{\frac{1-e^2}{e^2}}\arcsin e\right), \tag{7}$$

For example, if we examine a sphere, as a frequent case of an ellipsoid, we get $n_a = n_b = n_c = 1/3$ from (4), and the dipole moment assumes the familiar expression for the dipole moment of a sphere with radius a:

$$P_{c\phi} = E_0 \frac{a^3}{3} \frac{\varepsilon - 1}{1 + (\varepsilon - 1)^1/_3} = \frac{\varepsilon - 1}{\varepsilon + 2} a^3 E_0. \quad *$$

The components of the dipole moment along any axis of a biaxial spheroid can be computed from expressions (5), (6), and (7) and the scattered energy received by the radar station can be determined from this.

When the particles are smaller than the wavelength, i.e., when the Rayleigh law applies, the intensity of the scattered field is proportional to the square of the dipole moment. In this case, the particle radiates as an electric dipole with a moment determined by expression (2). The intensity of the scattered field will be proportional to the square of the particle volume and, in the case of a sphere, is proportional to the sixth power of its diameter.

Consequently, the components of the dipole moment of the particle along the chosen directions must be known in order to compute the field scattered by the particle.

The dipole moment of an ellipsoidal particle excited by a plane-polarized wave incident on it lies in the plane of polarization, if the latter coincides with one of the planes of symmetry of the ellipsoid. Therefore, when the electric vector of the incident plane-polarized wave is parallel to one of the axes of the ellipsoid, only the component of the dipole moment along this axis is excited. Correspondingly, the component of the dipole moment along the axis directed perpendicularly to the plane of polarization of the incident wave will not appear.

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* Here and in what follows the subscript **cф** stands for "sphere" (TR.)

Thus, if the radiated ellipsoidal particle is oriented such that one of its axes lies in the plane of polarization of the incident wave, the scattered wave will have the same plane of polarization as the incident wave. In this case the cross-polarized component will not appear in the scattered field. Consequently, depolarization can occur only if the particles are randomly oriented with respect to the plane of polarization of the exciting field.

2. DETERMINATION OF THE MAGNITUDE OF THE ECHO SIGNAL

Let the antenna of a radar station radiate in a certain direction z and let the ox-axis be oriented in the horizontal plane. Then the electric vector $\vec{E}$ will be in the xoy-plane and its direction will be fully defined if the angle is defined as the angle between $\vec{E}$ and the ox-axis. When the antenna radiates in a horizontal direction, the horizontal polarization corresponds to the angle $\alpha = 0^{\circ}$, and the vertical polarization to the angle $\alpha = 90^{\circ}$.

Let a system of coordinates ξ, η, ζ, associated with the scattering ellipsoidal particle, be introduced. Let the $o\eta$-axis and the $o\zeta$-axis be oriented along the equal axes of the ellipsoid and let the $o\xi$-axis be oriented along the axis of rotation. Then, e.g., the component of the dipole moment in the direction of the $o\xi$-axis will be determined by relation (2) with consideration of (5) for the case of a prolate spheroid and by an expression analogous to (2) with consideration of (6) for the symmetric $o\eta$- and $o\zeta$-axes.

The field components along the chosen axes ox, oy, and oz of the radar antenna must be found in order to calculate the energy received by the radar station.

Proceeding from (2), the components of the dipole moment along the axes of the ellipsoid will be:

$$P_\xi = \frac{abc}{3}\frac{\varepsilon - 1}{1 + (\varepsilon - 1) n} E_\xi = g E_\xi,$$

$$P_\eta = \frac{abc}{3}\frac{\varepsilon - 1}{1 + (\varepsilon - 1) n'} E_\eta = g' E_\eta,$$

$$P_\zeta = \frac{abc}{3}\frac{\varepsilon - 1}{1 + (\varepsilon - 1) n'} E_{n\zeta} = g' E_\zeta. \tag{8}$$

where $g = \frac{abc}{3}\frac{\varepsilon - 1}{1 + (\varepsilon - 1) n}$ and $g' = \frac{abc}{3}\frac{\varepsilon - 1}{1 + (\varepsilon - 1) n'}$ are introduced to abridge the notation.

Let the wave at point z = 0 have unit amplitude. Then the components fo the field radiated by the antenna will be:

$$E_x = \cos\alpha\, e^{\omega it},$$
$$E_y = \sin\alpha\, e^{\omega it},$$
$$E_z = 0. \tag{9}$$

For convenience in determining the dipole moment excited in the particle by the incident wave, let us introduce according to the familar system directional cosines between the systems ξ, η, ζ and x, y, z.

	x	y	z
ξ	α_1	α_2	α_3
η	β_1	β_2	β_3
ζ	γ_1	γ_2	β_3

Using the known correlations between the directional cosines and converting from the x, y, z system of coordinates to ξ, η, ζ, we find the following expressions for the components of the dipole moment excited in the particle:

$$P_{\xi} = g e^{\omega it} (\alpha_1 \cos\alpha + \alpha_2 \sin\alpha),$$
$$P_{\eta} = g' e^{\omega it} (\beta_1 \cos\alpha + \beta_2 \sin\alpha),$$
$$P_{\zeta} = g' e^{\omega it} (\gamma_1 \cos\alpha + \gamma_2 \sin\alpha).$$

(10)

Analogously, converting to the x, y, z coordinate system, we find the following components of the dipole moment at the surface of the antenna:

$$P_x = e^{\omega it} (g - g') \alpha_1 (\alpha_2 \sin\alpha + \alpha_1 \cos\alpha) + g' e^{\omega it} \cos\alpha,$$
$$P_y = e^{\omega it} (g - g') \alpha_2 (\alpha_2 \sin\alpha + \alpha_1 \cos\alpha) + g' e^{\omega it} \cos\alpha,$$
$$P_z = e^{\omega it} (g - g') \alpha_3 (\alpha_2 \sin\alpha + \alpha_1 \cos\alpha).$$

(11)

In particular, for a sphere

$$g = g' = g_{\text{сф}} = a^3 \frac{\varepsilon - 1}{\varepsilon + 2}$$

and the scattered energy, as expected, is polarized in the same plane as it is radiated.

The intensity of the scattered energy is proportional to the square of the dipole moment. Hence, in the general form, omitting the proportionality factors, the intensity of the scattered energy from the ellipsoidal particles is:

$$i_x = \overline{P_x^2},$$
$$i_y = \overline{P_y^2}.$$

(12)

With horizontal polarization ($\alpha = 0$), i_x is a parallel component of the scattering energy ($i_{\parallel}$) and it alone will be received by an ordinary radar station. The other term, i_y will give the cross-polarized component

($i_\perp$) which , as a rule, is not received by an ordinary radar station. Analogously, with vertical polarization ($\alpha = 90^\circ$), i_y will be the parallel component, and i_x the cross-polarized component of the scattered energy.

Later, we shall express the value of the energy scattered from the ellipsoidal particle in fractions of the energy scattered by a sphere of equal volume. This can be computed easily.

Having found the relationship of the squares of the corresponding amplitudes, we obtain:

$$I_x = \frac{i_x}{i_{c\phi}} = \frac{[(g-g')\,a_1(a_2 \sin\alpha + a_1 \cos\alpha) + g' \cos\alpha]^2}{g^2_{c\phi}},$$

$$I_y = \frac{i_y}{i_{c\phi}} = \frac{[(g-g')\,a_2(a_2 \sin\alpha + a_1 \cos\alpha) + g' \sin\alpha]^2}{g^2_{c\phi}}. \qquad (13)$$

For ellipsoids we introduce a shape factor ρ which characterizes the relationship of one of the ellipsoid diameters to the axis of rotation, i.e., e. $\rho = \frac{a}{b}$, if $a = c \neq b$. Table 1 shows the results of calculations of the g/g_{sp} and g'/g_{sp} values, which define the value of the energy received by the radar station for different values of ρ. The results of calculations are listed for water ($\epsilon = 81$) and ice ($\epsilon = 3$) particles.

ρ	$\frac{g}{g_{c\phi}}$	$\frac{g}{g_{c\phi}}$	ρ	$\frac{g}{g_{c\phi}}$	$\frac{g}{g_{c\phi}}$
			$\epsilon = 81$		
0,1	10,2	0,67	1,0	1,0	1,0
0,2	2,82	0,73	1,25	0,865	1,08
0,4	2,25	0,78	1,67	0,70	1,24
0,6	1,56	0,85	2,5	0,58	1,56
0,8	1,15	0,935	5,0	0,45	2,77
			$\epsilon = 3$		
0,1	1,6	0,84	1,0	1,0	1,0
0,2	1,45	0,86	1,25	0,94	1,03
0,4	1,3	0,89	1,67	0,855	1,09
0 6	1,17	0,93	2,5	0,77	1,17
0,8	1,05	0,97	5,0	0,66	1,33
1,0	1,09	1,0	10	0,61	1,46

Formula (13) and table 1 are initial expressions for further computations.

Generally speaking, the scattering particles in clouds can be randomly distributed with respect to the field radiated by the radar antenna. For practical purposes, it is important to investigate two special cases:

1) the particles are randomly oriented in space or in a plane and any position is equiprobable;

2) the particles have a specific, preferred orientation in space.

Of course, these special cases are not encountered in pure form in nature, because various factors (e.g., gravity) contribute to a specific orientation of the particles and other factors (e.g., small-scale turbulence) disorient them. However, when solutions are obtained for the first two special cases, it is not difficult to solve for any intermediate case, provided the degree of orientation of the particles in space is known.

3. RADAR SCATTERING BY NON-SPHERICAL RAINDROPS AND ORIENTED ICE PARTICLES

Experimental studies of falling drops have shown that large falling drops are not spherical, due to aerodynamic forces. Large raindrops usually approximate a biaxial oblate spheroid, whose axis of symmetry usually lies in the vertical plane. Correspondingly, the two major diameters of the drop are in the horizontal plane. Hence, in absence of a very strong wind, raindrops are specifically oriented in space so that, being equally distributed, they are larger in the horizontal plane and smaller in the vertical plane.

Thus, the axis of symmetry of the falling drop $o\xi$ will be vertical, and the $o\eta$-axis and the $o\psi$-axis will be horizontal.

Let us examine the energy received by a radar station which radiates a vertically polarized wave in a horizontal direction. Here

$$\alpha_1 = \cos(x, \xi) = 0,\ \alpha_2 = \cos(g, \xi) = 1,\ \alpha_3 \cos(z, \xi) = 0$$

and

$$I_x = 0,\ I_y = \left(\frac{g}{g_{c\phi}}\right)^2. \tag{14}$$

For horizontal polarization and the same direction of radiation, we get

$$I_x = \left(\frac{g'}{g_{c\phi}}\right),\ I_y = 0. \tag{15}$$

It is expedient to make analogous computations for ice particles of similar orientation. Figure 1 shows the results of such computations for water and ice particles when $\lambda = 3.2$cm. The component of scattering energy with vertical polarization is shown on the left and the component with horizontal polarization is shown on the right.

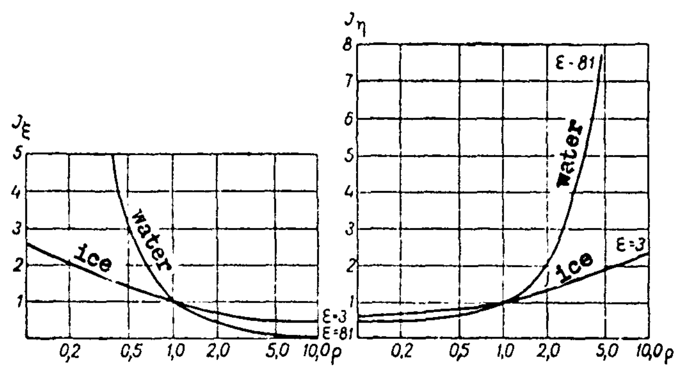

Figure 1. Relationship of the basic components of scattering energy to the shape factor ρ of water and ice particles. I_ξ is the component along the axis of rotation of the ellipsoid, I_η is the component in a perpendicular direction. The values of the scattered energy are given in fractions of the energy scattered by spherical particles of equivalent volume.

Figure 1 shows that the shape of a particle is especially significant in the case of water particles. For example, oblate water drops with $\rho=2$, i.e., drops which are twice as large horizontally as vertically and which can occur during heavy rains, scatter approximately 2.5 times less energy than spherical drops of equivalent volume in the case of vertical polarization, and 2 times more energy than spherical drops in the case of horizontal polarization.

4. SCATTERING OF RADIO WAVES BY ICE-CRYSTAL CLOUDS

We shall dwell briefly on the possibilities of using the evolved method for studying raidowave scattering by ice-crystal clouds.

Although non-sphericity exerts a weaker influence on scattering from ice particles, the effect of shape is still substantial, due to the large values for ice particles.

The shape factor will be especially great for the scattering of radio waves from melting crystalline particles. In this case the melting ice particle, maintaining its former shape which differs sharply from that of a sphere, is covered with a film of water, consequently, the particle will scatter and absorb electromagnetic energy as a completely uniform water particle. Quantitative analysis of the "bright band" phenomenon from the point of view of the decisive role of the shape of the particle found in the melting zone is outside the framework of this work. However, it should be noted that computations along this line, and analogous calculations and evaluations by Labrum [6] showed good agreement with experimental data.

In calculating the scattering values from clouds, it must be borne in mind that ice particles in a suspended state or moving but slightly can be randomly distributed in space and not have a definite orientation relative to the antenna of the radar station system. One may conceive of two actual cases:

1) prolate crystals in a suspended state randomly distributed in the horizontal plane, i.e., the arrangement of the crystal axes in the horizontal plane is arbitrary,

2) particles randomly distributed in all directions in space, due to the disturbing action of turbulent air currents, i.e., any position of the crystal in space is equiprobable.

The influence of the orientation of the particles on the power of the reflected signal is taken into account in expressions (11) and (13) by the variation of $\alpha_1\alpha_2\alpha_3$. Table 2 shows the required average values of $\alpha_1\alpha_2\alpha_3$. and their combinations in (11) and (13) calculated for both the cases indicated.

TABLE 2

	$\overline{\cos^2(x,\xi)}$	$\overline{\cos^2(y,\xi)}$	$\overline{\cos^4(x,\xi)}$	$\overline{\cos^4(y,\xi)}$	$\overline{\cos^2(x,\xi)\cos^2(y,\xi)}$
Random orientation of particles in the horizontal plane with the antenna directed upward........	1/2	1/2	3/8	3/8	1/8
Random orientation of particles in space........	1/3	1/3	1/5	1/5	1/15

Averaging relations (11) and (13), with consideration of the possible orientations, and using the result shown in table 2, we find that the intensity of the echo signal from particles randomly distributed in the horizontal plane with the antenna directed vertically upward is expressed by the equations

$$I_{\parallel} = \frac{1}{4} g^2_{c\phi}\left(\frac{3}{2} g^2 + gg' + \frac{3}{2} g'^2\right),$$

$$I_{\perp} = \frac{1}{8 g^2_{c\phi}} (g - g')^2. \tag{16}$$

The expressions (16) are correct for any polarization of the incident wave. Figure 2 shows the relationship between the echo-signal intensity and the particle shape.

Let us define the coefficient of depolarization as the ratio(in percents) of the cross-polarized component of scattering energy to the component of the energy scattered in the plane of the incident wave.

$$\overline{I}_{\parallel} = \frac{1}{5 g^2_{c\phi}}\left(g^2 + \frac{4}{3} gg' + \frac{8}{3} g'^2\right),$$

$$\overline{I}_{\perp} = \frac{1}{15 g^2_{c\phi}} (g - g')^2. \tag{17}$$

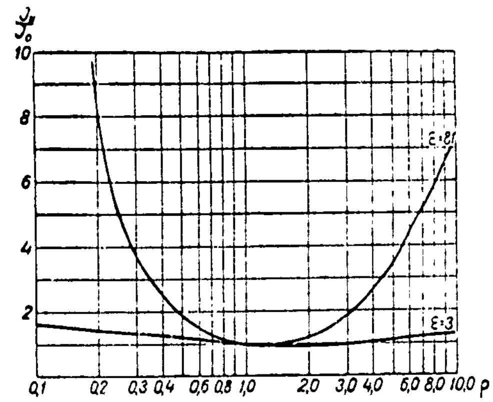

Figure 2. Relationship of the echo signal to the shape factor of water and ice particles randomly distributed in the horizontal plane. Values for the scattered energy are given in fractions of the energy scattered by spherical particles of equivalent volume.

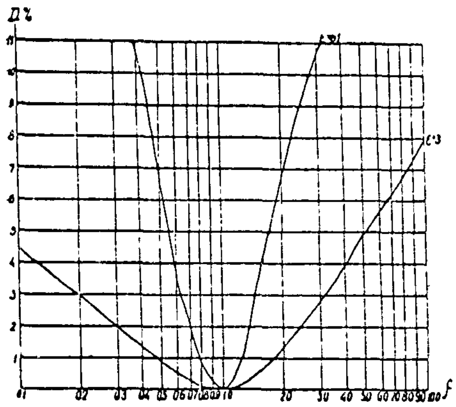

Figure 3. Value of the coefficient of depolarization with random distribution of ellipsoidal particles in the horizontal plane, as a function of the shape factor.

Figure 3 shows the relationship of the coefficient of depolarization to the shape of particles randomly distributed in the horizontal plane, viz., for rod-shaped particles ($\rho < 1$), whose axes of symmetry lie in the horizontal plane, i.e., in the plane perpendicular to the direction of radiation of the radar station. For comparison, the curve in this graph is continued into the ($\rho > 1$) region, which corresponds to disk-shaped particles.

$I_{\parallel}$ and $I_{\perp}$ can be obtained in a similar manner, given random spatial distribution of differently shaped particles, independently of the dire .tion of radiation in the plane of polarization.

Figures 4 and 5 show the value of the energy received by the radar station and by the coefficient of depolarization, given a random distribution of particles in space.

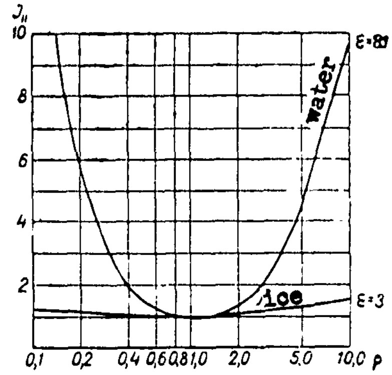

Figure 4. Relationship of the echo signal to the slope factor of particles randomly oriented in space. The scattered energy values are given as fractions of the energy scattered by spherical particles of equivalent volume.

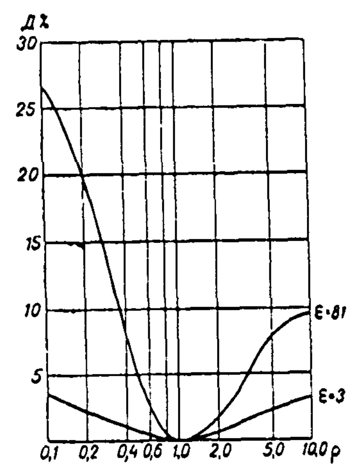

Figure 5. Value of the coefficient of depolarization, given random orientation of ellipsoidal particles in space, as a function of the shape factor.

After this work had been completed [6], we became acquainted with a recent book by Mason [4], who, referring to the data of Atlas, Kerker, and Hitschfeld [5], gives computational results for the special case discussed above and these results coincide closely with ours.

In both cases, randomly distributed non-spherical particles scatter considerably more energy than do spherical particles of equal volume, whence we conclude that precipitation and clouds consisting of randomly oriented non-spherical particles have great radar detectability. The effect for water or melting ice particles is especially large. Computations show that the signal can reach 10 decibels in the melting region, due solely to the non-sphericity of the ice particles of the cloud.

In the case of randomly distributed non-spherical particles (both in space and in a plane), an additional component of scattering energy $I_{\perp}$ will appear, i.e., the echo-signal component whose plane of polarization is perpendicular to the plane of polarization of the radiating field. The cross-polarized component will appear only in presence of non-spherical particles. For a sphere, $g = g'$, whence $I = 0$.

Although the cross-polarization component for an individual particle depends on its orientation and can be zero, on the whole, the average value of the cross-polarized component of the echo signal for a cloud or for a rain with random distribution of particles will not be zero and will sometimes reach tens of percents of the initial component.

An ordinary radar station receives a wave of the same polar. ation as the emitted wave. Therefore, an addit.onal signal cannot be detected in it. However, if provision is made for receiving the cross component, additional information about the target can be obtained. The shape of the particles can be determined for a given orientation or for random orientation of the particles, or, conversely, their orientation with respect to the sounding beam can be determined.

The echo signal from ice-crystal clouds can be rather high, due to the relatively large size of ice particles. Thus, in calculating the echo signal from non-spherical particles, one may use the well developed technique for calculating the echo signal from spherical particles.

The method of calculating the echo signal from ice-crystal clouds is based on calculation of the echo signal from spherical particles of the same volume and subsequent calculation of their shape and orientation by means of the graphs in figs. 1, 2, and 4.

The calculations show that the shape factor must be introduced for large-drop water precipitation and for water-filled and oriented ice particles. If the ice crystals are randomly oriented, the shape factor will be near unity and the echo signal can be calculated in the same manner as for ordinary drop clouds.

As an example, let us consider the radar reflectivity of an ice-crystal cloud of the Ci or Cs type composed of columnar crystals randomly oriented in space and 0.1 mm long, 0.02 mm thick, with a concentration of 10^{-1} cm^{-3} [8].

For spherical particles, the radar reflectivity (with consideration of dielectric permeability) is:

$$S_{\text{сф}} = \sum_{i=1}^{\infty} N_i d_i^6 \left(\frac{\varepsilon - 1}{\varepsilon + 2}\right)^2. \tag{18}$$

Assuming a nearly ellipsoidal cloud crystal, let us find the diameter of a sphere of equal volume

$$d_{\text{сф}} = \sqrt[3]{8ab^2} = 3{,}4 \cdot 10^{-2}\ \text{мм}.$$

Substituting in (18), we get $S = 2.6 \times 10^{-17}$ cm^3.

Given a random distribution of particles, the excess of signal power in comparison to equidimensional spherical particles will not be great. When $\rho = 0.2$, from fig. 2 we get $I \approx 1.5$.

Consequently, the radar reflectivity of ice-crystal clouds, given the above conditions, will be

$$S_{cr_{\parallel}} = 4 \cdot 10^{-17} \text{ cm}^3$$

and the depolarization component will correspond to a cloud with radar reflectivity (fig. 3):

$$S_{\perp} = 10^{-18} \text{ cm}^3.$$

Thus the radar reflectivity of the parallel component of a pure ice cloud is close to the reflectivity of water clouds detected by radar: however, the sensitivity of the radar device must be increased by a factor of 1-1.5 in order to detect the cross-polarized component of the scattered energy.

5. RELATIONSHIP OF RADAR SCATTERING BY NON-SPHERICAL PARTICLES TO THE TILT ANGLE OF THE ANTENNA

Given a definite particle orientation, scattering will depend on the direction of the plane of polarization and, consequently, on the vertical tilt angle of the antenna. Earlier we examined a special case of horizontal radiation of a radar station and of a vertical arrangement of the axes of symmetry of the scattering particles. This applies to scattering from raindrops, given a horizontal antenna direction. In the general case of vertical cross sections, scattering will be a function of the tilt angle of the antenna.

Let δ be the angle between the direction of radiation (axis z) and the vertical, and let ψ be the angle in the horizontal plane between the axis of rotation ξ and direction x. Then, given a vertical orientation of the particles ($o\xi$ is vertical) we get

$$\cos(x,\xi) = 0, \ \cos(y,\xi) = \sin\delta. \tag{19}$$

Consequently, the value of the scattered energy is this case will be

$$\begin{aligned} I_{\parallel} &= \frac{1}{g^2_{c\phi}} [(g-g')^2 \sin^4\delta + (g-g')\, g' \sin^2\delta + g'^2, \\ I_{\perp} &= 0. \end{aligned} \tag{20}$$

Figure 6 shows the relationship of scattering to the tilt angle of the antenna, given vertical polarization and vertically oriented ice particles. As can be seen from the graph, scattering from ice crystals with a form characterized, e. g., by the factor $\rho = 0.2$, changes 5-6 fold when the tilt angle of the antenna is changed from 0° to 90°.

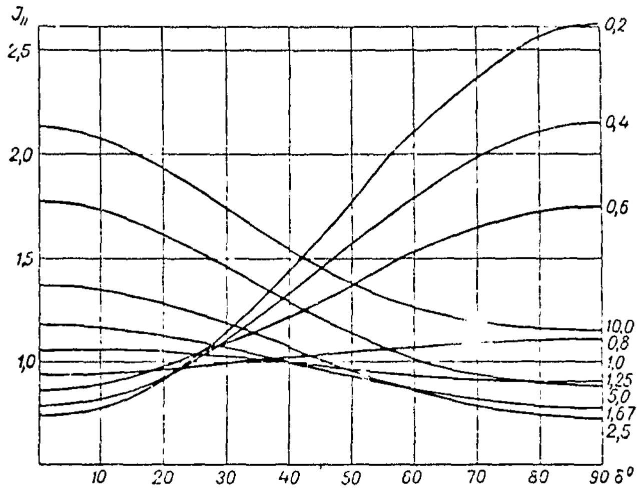

Figure 6. Relationship of the value of the signal received by radar to the tilt angle of the antenna, given vertical polarization. The figures at the ends of the curves indicate the value of the shape factor ρ. The scattered energy value is given in fractions of the energy scattered by spherical particles of equivalent volume.

Given a horizontal particle orientation, we get the following when the axis of symmetry is randomly oriented in the horizontal plane:

$$\cos(x, \xi) = \cos\psi, \quad \cos(y, \xi) = \sin\psi\cos\delta. \tag{21}$$

whence, for vertical polarization, we get

$$I_{\parallel} = \frac{1}{g'^{2}_{c\psi}}\left[\frac{3}{8}(g - g')^{2}\cos^{4}\delta + (g - g')g'\cos^{2}\delta + g'^{2}\right],$$
$$I_{\perp} = \frac{1}{8}(g - g')^{2}\cos^{2}\delta. \tag{22}$$

In this case, a change of the tilt angle does not have as great an effect as in the case of vertical orientation of the particles, but an additional depolarization component ($I_{\perp}$) appears, which is maximum when the antenna is vertical.

In the case of vertical polarization: if the main signal changes while the antenna is being tilted and if a depolarization component is detected, the scattering non-spherical particles are entirely in the horizontal plane. If a depolarization component does not appear under the same conditions, the particles are in the vertical plane. If the depolarization component is present and no changes are detected in the main signal while the antenna is being tilted, the non-spherical particles are randomly oriented in space.

Let us examine the relationship of the echo signal to the tilt angle of the antenna in a similar manner, given horizontal polarization ($\alpha = 0$)

If the axes of rotation of the particles lie in the vertical plane, we get

$$I_{\parallel} = \frac{g'^2}{g^2_{c\phi}},$$
$$I_{\perp} = 0 \tag{23}$$

and if they are randomly oriented in the horizontal plane:

$$I_{\parallel} = \frac{1}{g^2_{c\phi}}\left[\frac{3}{8}(g - g')^2 + (g - g')g' + g'^2\right],$$
$$I_{\perp} = \frac{1}{8g^2_{c\phi}}(g - g')^2 \cos^2 \delta. \tag{24}$$

As can be seen from (23) and (24), when a radar station radiates a horizontally polarized plane wave, scattering from non-spherical particles will not depend on the direction of radiation of the station. The cross-polarized component will not appear in this case when the axes of symmetry of the particles are positioned vertically, and will be proportional to $\cos^2 \delta$ if they are randomly oriented in the horizontal plane.

Consequently, the appearance of a depolarization component during the tilting of an antenna which radiates a horizontally polarized wave attests to the horizontal arrangement of the non-spherical particles. On the other hand, the absence of a depolarization component, given a constant main reflected signal, indicates that the axes of symmetry of the non-spherical scattering particles are positioned vertically. In addition to the practical considerations indicated above, the following important conclusions can be drawn from the results obtained:

1) for detection of ice-crystal clouds usually consisting of horizontally oriented ice particles, it is expedient to use horizontal polarization instead of the usual vertical polarization;

2) the echo signal from ice-crystal clouds should increase when the antenna is directed vertically upward;

3) The character of the reflecting particles can be determined by changing the angle between the direction of polarization of the radiated and received wave and by observing the resultant change in the echo signal; if there are no regular changes in the intensity of the echo signal, the particles are either spherical or they are randomly oriented. The latter can be established by the appearance of the depolarization component.

Similar results may also be obtained by changing the angle of elevation of the antenna, if the concentration, dimensions, and orientation of the particles are regarded as constant in the target regions of the cloud.

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